

Spontaneous CP and R parity breaking in supersymmetry

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We show that a model where both CP and R-parity are spontaneously broken exists. We study the electroweak symmetry breaking sector of the model and find minima consistent with experimentally viable Higgs boson masses. We also demonstrate that one can obtain neutrino masses and mixing angles within measured values.

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Breaking CP in the Higgs sector, either spontaneously or explicitly, is a theoretically attractive alternative to the CP violating phase present in the Standard Model (SM) [1, 2]. New sources for CP violation from beyond the Standard Model scenarios are needed to generate enough CP violation to explain the matter-antimatter asymmetry in the universe. While in supersymmetric models a large number of new phases emerge, in a general minimal supersymmetric standard model (MSSM) these phases are strongly constrained by electric dipole moments [3].

A defect of the Standard Model, which persists in the MSSM, is that the neutrino masses vanish. Yet the neutrino experiments have provided strong evidence for small nonvanishing neutrino mass [4]. One popular way to explain the neutrino masses is a small violation of R-parity [5], $R_p = (-1)^{3B-L+2s}$, where B =baryon number, L =lepton number, and s =spin of the particle. Another is the seesaw mechanism [6] which can generate small masses for neutrinos by allowing Majorana masses through the presence of heavy right-handed neutrinos.

Supersymmetric models share the problem of origin of CP violation with the Standard Model. In addition, there is no fundamental reason for the existence of R-parity in the MSSM, where it is put in by hand in order to protect the proton from decaying. However, if only lepton number (or baryon number) is violated, the proton does not decay.

Both neutrino masses and CP violation could be explained if CP and R were symmetries of the Lagrangian, but spontaneously broken by the vacuum. In this letter we provide a model where both of these violations are intertwined and both are spontaneous. We show that while it is non-trivial to satisfy conditions for both symmetry breakdowns at the same time, there are regions in the parameter space where we can realize suitable Higgs masses, as well as measured neutrino mass differences and mixing angles.

A model where both favored neutrino mass genera-

tion mechanisms - namely seesaw and R-parity violation - with spontaneous R-parity violation was realized in [7], where the spontaneous R_p violation was introduced via a term proportional to NLH_2 . This represents the familiar bilinear R_p violating term mixing lepton and Higgs superfields, LH_2 , when the right-handed sneutrino field, \tilde{N} , develops a VEV. This term also breaks the lepton number spontaneously (but not R-parity), and thus introduces a superfluous massless Goldstone boson into the scalar spectrum. The problem can be solved by adding a singlet S to the theory, through a term N^2S , which explicitly breaks lepton number, as S cannot be assigned lepton number -2 .

Attempts to violate CP spontaneously, by complex VEVs of the neutral scalars exist [1], but fulfilling the experimental constraints has proven difficult. More than one Higgs doublet is needed, see *e.g.* [8]. Spontaneous breaking of CP is not possible at tree level in the MSSM with two Higgs doublets, while it is allowed in a model with three doublets [9]. Instead of adding doublets, one can study extended models, like the NMSSM model [10], where the so called μ -problem has been avoided by adding a singlet and requiring \mathbf{Z}_3 symmetry. At tree-level one cannot get spontaneous CP violation in this model either and consequently radiative corrections were evoked [11]. In that case a very light Higgs boson emerges [12], as also happens in the MSSM, if spontaneous CP violation is induced via radiative correction [13]. Another possibility studied is to discard the \mathbf{Z}_3 symmetry completely. On one hand, this way one loses the solution to the μ problem, on the other hand, it is possible to achieve SCPV [14] and also solve the problem of domain walls, which are created during the EW phase transition as the \mathbf{Z}_3 symmetry is broken spontaneously.

An interesting model for spontaneous CP violation was presented in [15], where the \mathbf{Z}_3 symmetry is replaced by R-symmetries on the whole superpotential, including non-renormalizable terms [16]. The method generates a tadpole term for the singlet field S in the soft SUSY

breaking part of the Lagrangian. This tadpole term allows for spontaneous CP violation to occur at tree-level [15, 17]. The tadpole is assumed to originate from non-renormalizable interactions, which do not spoil quantum stability. We adopt this approach here.

The superpotential of our model is

$$W = h_U^{ij} Q_i H_2 U_j - h_D^{ij} H_1 Q_i D_j - h_E^{ij} H_1 L_i E_j \\ + h_N^{ij} L_i H_2 N_j + \lambda_H H_1 H_2 S + \frac{\lambda_S}{3!} S^3 + \frac{\lambda_{N_i}}{2} N_i^2 S,$$

where H_1 and H_2 denote the Higgs doublet superfields, L_i and Q_i the left-handed lepton and quark doublet superfields, respectively, and E_i and U_i , D_i the lepton and quark singlet superfields. Right-handed neutrino superfields are denoted by N_i and S is the gauge singlet superfield. The terms in the Lagrangian are the only renormalizable ones that respect CP and R -parity, in addition to the gauge symmetry. As all the parameters in the Lagrangian are real, this solves the strong CP problem [8]. The only possible global symmetries are the baryon number and \mathbf{Z}_3 .

The soft SUSY breaking terms in this model are the mass terms for scalars and gauginos, and the part mirroring the superpotential with an additional S -tadpole,

$$V_{soft} = \left[A_U^{ij} \tilde{Q}_i H_2 \tilde{U}_j - A_D^{ij} H_1 \tilde{Q}_i \tilde{D}_j - A_E^{ij} H_1 \tilde{L}_i \tilde{E}_j \right. \\ \left. + A_N^{ij} \tilde{L}_i H_2 \tilde{N}_j + A_H H_1 H_2 S + \frac{A_S}{3!} S^3 + \frac{A_{N_i}}{2} \tilde{N}_i^2 S \right. \\ \left. - \xi^3 S + h.c. \right] + M_\phi^2 \phi_i^* \phi_j + M_a(\lambda_a^2 + \lambda_a^{*2}).$$

Here ϕ runs over the scalar fields, i, j over the possible family indices and a over the three gauge groups. In calculations, we take the mass matrices to be flavor diagonal, and assume that only the third generation Yukawa couplings (except those for neutrinos) differ from zero. We then impose the same texture on the corresponding A -terms, and treat the neutrino Yukawa couplings as free parameters. The full tree-level scalar potential is $V_s = V_{soft} + V_F + V_D$, where V_F and V_D are the usual F and D terms.

The minimization of the scalar potential with respect to the fields ϕ_i and the corresponding phases θ_i yield constraints later used in finding the scalar mass matrix,

$$\left. \frac{\partial V_s}{\partial \phi_i} \right|_{\phi=\langle \phi \rangle} = 0, \quad \left. \frac{\partial V_s}{\partial \theta_i} \right|_{\phi=\langle \phi \rangle} = 0. \quad (1)$$

Without spontaneous CP violation, the VEVs are real and the minimization equations with respect to the phases are always satisfied.

The minimization equations for the charged scalars can be trivially solved by setting all charged scalar VEVs to zero. As long as the tree-level masses of these fields remain positive and the corresponding soft A -terms remain small enough, this is also the global minimum of

the potential with respect to these fields [7]. We use the seventeen equations (the phase of the H_1 field can always be rotated away) given by the moduli and phases of the neutral scalar fields to solve for the soft masses of the corresponding fields and subset of A -parameters ($A_N, A_S, A_{N_i}, A_N^{i,3}$). Thus the complex VEVs remain free parameters and we denote

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\delta_2} \end{pmatrix}, \quad \langle S \rangle = \sigma_S e^{i\theta_S}, \\ \langle \tilde{L}_i \rangle = \begin{pmatrix} \sigma_{L_i} e^{i\theta_{L_i}} \\ 0 \end{pmatrix}, \quad \langle \tilde{N}_i \rangle = \sigma_{R_i} e^{i\theta_{R_i}}. \quad (2)$$

Note that since R_p is violated, the W mass is $m_W^2 = \frac{1}{2} g_2^2 v^2$ where $v^2 \equiv v_1^2 + v_2^2 + \sigma_{L_i}^2 \approx (174 \text{ GeV})^2$.

Writing the fields as $\phi \equiv \phi_r + i\phi_i$, with nine neutral scalar fields (two Higgs, one singlet and six sneutrinos) we get an 18×18 dimensional mass matrix for the scalars. The radiative corrections to the scalar masses are implemented via the one-loop effective scalar potential,

$$V_{1-loop} = \frac{-3}{32\pi^2} \left[m_{\tilde{t}_1}^4 \log \left(\frac{m_{\tilde{t}_1}^2}{\Lambda^2} - \frac{3}{2} \right) \right. \\ \left. + m_{\tilde{t}_2}^4 \log \left(\frac{m_{\tilde{t}_2}^2}{\Lambda^2} - \frac{3}{2} \right) - 2m_t^4 \log \left(\frac{m_t^2}{\Lambda^2} - \frac{3}{2} \right) \right],$$

where Λ is the renormalisation scale. A similar term for the bottom (s)quark masses contributes significantly, if $\tan \beta$ is large. Here $m_t^2 = y_t v^2$ and $m_{\tilde{t}_i}^2$ are the eigenvalues of the stop mass matrix,

$$M_{\tilde{t}_{L\tilde{L}}}^2 = M_{Q_{33}}^2 + m_t^2 - \left(\frac{g_1^2}{12} - \frac{g_2^2}{4} \right) (v_1^2 - v_2^2 + \sigma_{L_i}^2), \\ M_{\tilde{t}_{L\tilde{R}}}^2 = A_t v_2 e^{-i\delta_2} \\ + y_t (v_1 \lambda_H \sigma_S e^{i\theta_S} + h_N^{ij} \sigma_{L_i} \sigma_{R_j} e^{i(\theta_{L_i} - \theta_{R_j})}), \\ M_{\tilde{t}_{R\tilde{R}}}^2 = M_{U_{33}}^2 + m_t^2 + \frac{g_1^2}{3} (v_1^2 - v_2^2 + \sigma_{L_i}^2). \quad (3)$$

The loop corrections lead to additional terms in both the minimisation conditions and the scalar mass matrix. In numerical calculations, we omit the D -term contributions and set for simplification, $M_{Q_{33}} = M_{U_{33}} = M_{SUSY}$, with $M_{SUSY} \sim \Lambda \sim 1 \text{ TeV}$.

The following experimental input is used

$$v^2 = 174 \text{ GeV}, \quad m_W = 80.42 \text{ GeV}, \quad m_t^{Pole} = 180 \text{ GeV}, \\ \alpha_s = 0.102, \quad m_\tau = 1.777 \text{ GeV}, \quad \sin^2 \theta_w = 0.23124 \quad (4)$$

The rest of our free parameters are randomly sampled, with sampling ranges as follows (the couplings λ_i are constrained by perturbativity):

$$0.1 < \lambda_{H,N_i} < 0.4, \quad 0.2 < \lambda_S < 0.7, \quad |h_N| < 10^{-7}, \\ 0.4 \text{ TeV} < \xi < 1 \text{ TeV}, \quad -\pi < \theta_\phi < \pi, \quad |\langle S \rangle| < 1 \text{ TeV}, \\ |\langle \tilde{L}_i \rangle| < 100 \text{ keV}, \quad |\langle \tilde{N}_i \rangle| < 1 \text{ TeV}, \quad 2 < \tan \beta < 60, \quad (5)$$

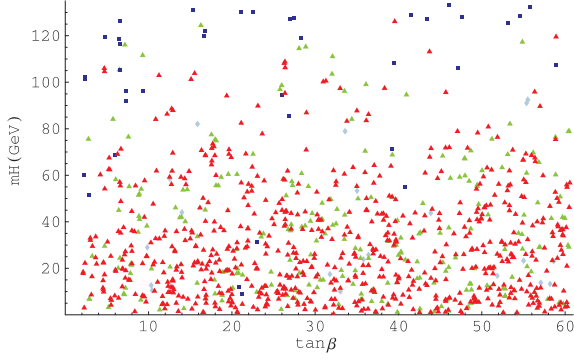


FIG. 1: Lightest physical scalar mass as a function of $\tan \beta$. The main component of the lightest scalar is $H_{1,2}^0$ (blue rectangle), S (light blue diamond), $\tilde{\nu}_L$ (green triangle), or \tilde{N} (red triangle).

and the A -parameters not eliminated by Eq.(1) vary between $0 < A_N^{ij} < (1 \text{ TeV})h_N^{ij}$.

In the limit $\lambda_{N_i} \rightarrow 0$ we recover lepton number conservation. CP and R-parity are still spontaneously violated, and thus in this limit the model contains an experimen-

tally disallowed Goldstone boson.

There are a total of 55 relevant parameters $\langle \sigma_\phi \rangle$, θ_ϕ , λ_S , λ_H , λ_{N_i} , h_N^{ij} , M_ϕ^2 , A_S , A_H , A_{N_i} , A_N^{ij} and ξ . 17 of these can be eliminated by using the minimization conditions, leaving 38 free parameters. Sampling over such a large space makes it extremely difficult to find viable minima of the scalar potential, even without applying any other cuts or considerations. For the full model we choose to set $\sigma_{R_{1,2}} = \theta_{R_{1,2}} = 0$ to further reduce the sampling space. This choice identically solves the vacuum conditions $\partial_{\theta_{R_1}} V = 0$ and $\partial_{\theta_{R_2}} V = 0$.

In this model, the couplings of the lightest scalar are reduced compared to the SM, due to the (sometimes dominant) singlet components. Thus the corresponding experimental limits are smaller as well than the experimental lower bound for the SM Higgs boson, $m_H \gtrsim 114 \text{ GeV}$, see e.g. discussion in [18]. In Fig. 1 we plot the mass of the lightest Higgs boson as a function of $\tan \beta$, and show that experimentally acceptable Higgs bosons masses can be found for a large part of the parameter space.

In a field basis of ν_{L_i} , N_i , \tilde{S} , \tilde{H}_1^0 , \tilde{H}_2^0 , λ_0 , λ_3 the neutral fermions form the following 11×11 mass matrix:

$$M_{\chi^0} = \begin{pmatrix} \mathbf{0}_{3 \times 3} & \mathbf{h}_N^{3 \times 3} \langle H_2^0 \rangle & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & h_N^{i,j} \langle \tilde{N}_j^* \rangle & -\frac{g_1}{\sqrt{2}} \langle \tilde{\nu}_{L_i}^* \rangle & \frac{g_2}{\sqrt{2}} \langle \tilde{\nu}_{L_i}^* \rangle \\ \mathbf{h}_N^{3 \times 3} \langle H_2^0 \rangle & \mathbf{1}_{3 \times 3} \lambda_{N_i} \langle S \rangle & \lambda_{N_i} \langle \tilde{N}_i^* \rangle & 0 & h_N^{j,i} \langle \tilde{\nu}_{L_j} \rangle & 0 & 0 \\ \mathbf{0}_{1 \times 3} & \lambda_{N_i} \langle \tilde{N}_i^* \rangle & \lambda_S \langle S \rangle & \lambda_H \langle H_2^0 \rangle & \lambda_H \langle H_1^0 \rangle & 0 & 0 \\ \mathbf{0}_{1 \times 3} & 0 & \lambda_H \langle H_2^0 \rangle & 0 & \lambda_H \langle S \rangle & -\frac{g_1}{\sqrt{2}} \langle H_1^0 \rangle & \frac{g_2}{\sqrt{2}} \langle H_1^0 \rangle \\ h_N^{i,j} \langle \tilde{N}_j^* \rangle & h_N^{j,i} \langle \tilde{\nu}_{L_j} \rangle & \lambda_H \langle H_1^0 \rangle & \lambda_H \langle S \rangle & 0 & \frac{g_1}{\sqrt{2}} \langle H_2^{0*} \rangle & -\frac{g_2}{\sqrt{2}} \langle H_2^{0*} \rangle \\ -\frac{g_1}{\sqrt{2}} \langle \tilde{\nu}_{L_i}^* \rangle & 0 & 0 & -\frac{g_1}{\sqrt{2}} \langle H_1^0 \rangle & \frac{g_1}{\sqrt{2}} \langle H_2^{0*} \rangle & M_1 & 0 \\ \frac{g_2}{\sqrt{2}} \langle \tilde{\nu}_{L_i}^* \rangle & 0 & 0 & \frac{g_2}{\sqrt{2}} \langle H_1^0 \rangle & -\frac{g_2}{\sqrt{2}} \langle H_2^{0*} \rangle & 0 & M_2 \end{pmatrix} \quad (6)$$

It is easy to see the structure of the usual seesaw mechanism, which produces small neutrino masses m_ν ,

$$M_{\chi^0} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}, \quad m_\nu = -m_D M_R^{-1} m_D^T, \quad (7)$$

where $m_D \ll M_R$. Similarly to [7], there are actually several sources for neutrino masses: the usual seesaw and the mixing of neutrinos with \tilde{h}_2^0 and gauginos.

Inspecting the requirement that $m_D \ll M_R$ yields some qualitative understanding of the model. In particular, the left-handed sneutrino VEVs must be small and $h_N \langle \tilde{N}^* \rangle$ should be of the same order. Thus, although $\langle \tilde{N}^* \rangle$ is not bound by any other prior consideration, having $h_N \approx 10^{-7}$ results in an upper limit of a few TeV for the right-handed sneutrino VEVs.

We diagonalise M_{χ^0} numerically and use $M_1 \sim M_2 \sim 1 \text{ TeV}$. Great care must be taken, as the elements of M_{χ^0} may vary over ten orders of magnitude, and the eigen-

values themselves over as much as twenty orders of magnitude. The diagonalising matrix \mathcal{N} , with $\mathcal{N} M_{\chi^0} \mathcal{N}^{-1} = \text{diag}(m_{\chi_i^0}, m_{\nu_j})$, has the following general form

$$\mathcal{N} = \begin{pmatrix} \zeta & N_\chi \\ V_\nu^T & \bar{\zeta}^T \end{pmatrix}. \quad (8)$$

Here $\zeta, \bar{\zeta} \ll 1$ denote 8×3 matrices that can be determined perturbatively, see e.g. [19]. Our interest lies in the matrix V_ν , the neutrino mixing matrix. Using the canonical notation for the neutrino mixing matrix [19], we can extract the mixing angles as follows

$$\begin{aligned} \sin \theta_{13} &= |V_\nu^{13}| = |\mathcal{N}^{11 \ 1}|, \\ \tan \theta_{12} &= \left| \frac{V_\nu^{12}}{V_\nu^{11}} \right| = \left| \frac{\mathcal{N}^{10 \ 1}}{\mathcal{N}^{9 \ 1}} \right|, \\ \tan \theta_{23} &= \left| \frac{V_\nu^{23}}{V_\nu^{33}} \right| = \left| \frac{\mathcal{N}^{11 \ 2}}{\mathcal{N}^{11 \ 3}} \right| \end{aligned} \quad (9)$$

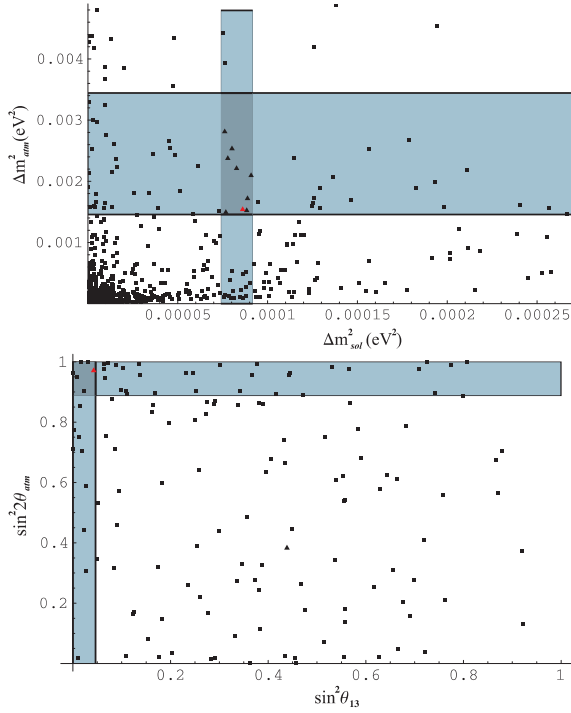


FIG. 2: Correlations of neutrino angles and mass differences. Experimentally allowed regions at 3σ level are colored. Points satisfying neutrino mass constraints are marked as triangles. Upper plot: Δm_{atm}^2 vs Δm_{sol}^2 , lower plot: $\sin^2 2\theta_{23}$ vs $\sin^2 \theta_{13}$, requiring at the same time that the experimental limit for $\sin^2 \theta_{12}$ is satisfied. One point satisfies all constraints (red).

Once we have sufficient viable minima, i.e. points in parameter space that produce a minimum of the potential, we apply experimental constraints. We calculate how much each minimum deviates from the current experimental results and select a number of closest events for further analysis. The experimental input we use is the following [20]:

$$\begin{aligned} \sin^2 2\theta_{23} &\geq 0.89, \quad \sin^2 \theta_{13} \leq 0.047, \\ \sin^2 \theta_{12} &\simeq 0.23 - 0.37, \\ \Delta m_{atm}^2 &\simeq 1.4 \times 10^{-3} \text{eV}^2 - 3.3 \times 10^{-3} \text{eV}^2, \\ \Delta m_{sol}^2 &\simeq 7.3 \times 10^{-5} \text{eV}^2 - 9.1 \times 10^{-5} \text{eV}^2. \end{aligned} \quad (10)$$

The first stage of finding points for the full model which are minima of the scalar potential has a signal-to-noise ratio of one over fifty thousand, which shows the odds of having an event fall within the constraints. Of these points only roughly one in a thousand will satisfy all the above constraints. In Fig. 2 we demonstrate that it is possible to obtain models where the mass differences and experimentally found mixing angles are satisfied.

Summarizing, we have shown that a model that violates both CP invariance and R-parity exists, and we constructed it explicitly. We have shown that experimentally viable neutrino and Higgs boson masses can be obtained. We have reason to expect that in our model EDM bounds and experimental results on kaon (especially ϵ_K) and B physics can be satisfied [21].

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